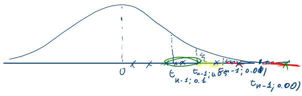
$$\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$$

$\sigma^2$ INCOGNITA

$$SE(\hat{\mu}) = \frac{\sigma}{\sqrt{n}}$$

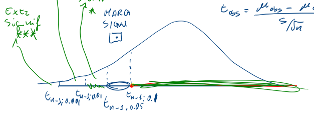
$$\frac{\hat{\mu}_{obs} - \mu_0}{S/\sqrt{n}} \sim t_{n-1} \qquad \widehat{SE(\hat{\mu})} = \frac{S}{\sqrt{n}}$$

$$S = \sqrt{\frac{n}{n-1}}\,\hat{\sigma}$$




$$\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$$

$$t_{obs} = \frac{\hat{\mu}_{obs} - \mu_0}{S/\sqrt{n}}$$

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$\mu_0 = 50$ mg $\qquad \hat{\sigma} = 2.5$ mg

$n = 25$

$$\begin{cases} H_0: \mu = \mu_0 = 50\,mg \\ H_1: \mu \neq \mu_0 = 50\,mg \end{cases}$$

(A) FORMULAZIONE HP

(B) Scelta e calcolo STAT. TEST

$$S = \sqrt{\frac{n}{n-1}} \cdot \hat{\sigma} = \sqrt{\frac{25}{25-1}} \cdot 2.5 = 2.552$$

$$\frac{\hat{\mu} - \mu_0}{S/\sqrt{n}} \sim t_{n-1}$$

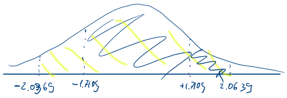
$$t_{obs} = \frac{51 - 50}{2.552/\sqrt{25}} = 1.96$$

(C) CONCLUSIONE $\rightarrow H_1$ bilat.

$\Rightarrow \alpha/2 \qquad \alpha = 0.1,\ 0.05,\ 0.01,\ 0.001$

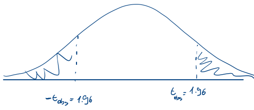
$\alpha/2 = 0.05,\ 0.025,\ 0.005,\ 0.0005$

$t_{25-1;0.05} = 1.7109 \qquad t_{25-1;0.025} = 2.0639$




Essendo $\quad 1.7109 < t_{obs} < 2.0639$

$\Rightarrow$ DIFF. MARGINALMENTE SIGNIFICATIVA




$$P_{value} = P(|T_{25-1}| > |1.96|) = 0.06756$$

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se $\hat{\theta}$ è lo SMV per $\theta$

se $n$ abbastanza grande

$$\hat{\theta} \sim N(\theta;\ SE^2(\hat{\theta}))$$

$$H_0: \theta = \theta_0$$

$\Rightarrow$ sotto $H_0 \qquad \hat{\theta} \sim N(\theta_0;\ SE^2(\hat{\theta}))$

sotto $H_0$

$$Z = \frac{\hat{\theta} - \theta_0}{SE(\hat{\theta})} \sim N(0,1)$$

$$T = \frac{\hat{\mu} - \mu_0}{S/\sqrt{n}} \xrightarrow[n \to +\infty]{} N(0,1)$$

TEST PER $\pi$

siano $X_1, \dots, X_n$ $n$ VC IID

$X_i \sim Ber(\pi)$

$$\hat{\pi} = \frac{1}{n}\sum_{i=1}^{n} X_i \underset{a}{\sim} N\left(\pi;\ \frac{\pi(1-\pi)}{n}\right)$$

$$\begin{cases} H_0: \pi = \pi_0 \\ H_1: \text{ALTERNAT.} \end{cases}$$

sotto $H_0$

$H_0: \pi = \frac{1}{2}$

$$\hat{\pi} \underset{a}{\sim} N\left(\pi_0;\ \frac{\pi_0(1-\pi_0)}{n}\right)$$

$$Z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$$

$$z_{obs} = \frac{\hat{\pi}_{obs} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

$n = 50$ 30 successi su 50 lanci

$\pi_0 = 0.5$

$$\begin{cases} H_0: \pi = \pi_0 = 0.5 \\ H_1: \pi > \pi_0 = 0.5 \end{cases}$$

TEST z PER $\pi$

$$\hat{\pi} = \frac{30}{50} = 0.6$$

$\boxed{A}$ Hp

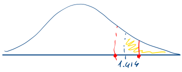
$$\begin{cases} H_0: \pi = \pi_0 = \frac{1}{2} \\ H_1: \pi > \pi_0 = \frac{1}{2} \end{cases}$$

$\boxed{B}$ scelta e calcolo

test z per binom

$$\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \underset{a}{\sim} N(0,1)$$

$$z_{obs} = \frac{0.6 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{50}}} = 1.414$$




$$\text{Pvalue} = P(Z > 1.414) = 1 - \Phi(1.414) = 0.079$$

$0.05 < \text{Pvalue} < 0.1 \implies$ MARG. SIGNIF.

60 successi su $n = 100$

$$\hat{\pi} = \frac{60}{100} = 0.6$$

$\boxed{A}$

$$\begin{cases} H_0: \pi = 0.5 \\ H_1: \pi > 0.5 \end{cases}$$

$\boxed{B}$

$$\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$$

$$z_{obs} = \frac{0.6 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{100}}} = 2$$

$$\text{Pvalue} = P(Z > 2) = 1 - \Phi(2) = 0.0228$$

$$0.01 < 0.0228 < 0.05$$

$$\hat{\pi} = \frac{600}{1000} = 0.6$$

$$z_{obs} = \frac{0.6 - 0.5}{\sqrt{\frac{0.5(1-0.5)}{1000}}} = 6.325$$

$$P(Z > 6.325) = 1 - \Phi(6.325) = 1e-10$$

$$\approx 1 - 1 = 0.$$

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$n = 100$ $S_{100} = 30$ $\pi_0 = 0.4$

$\boxed{A}$

$$\begin{cases} H_0: \pi = \pi_0 = 0.4 \\ H_1: \pi < \pi_0 = 0.4 \end{cases}$$

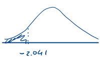
$$\hat{\pi}_{obs} = \frac{30}{100} = 0.3$$

$\boxed{B}$

$$\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$$

$$z_{obs} = \frac{0.3 - 0.4}{\sqrt{\frac{0.4(1-0.4)}{100}}} = -2.041$$

$\boxed{C}$




$$\text{Pvalue} = P(Z < -2.041) = 1 - \Phi(2.041) = 0.0206$$

$0.01 < \text{Pvalue} < 0.05 \implies$ SIGNIF. *

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$n = 100$ $S_{100} = 20$ $\pi_0 = 0.25$

$\boxed{A}$

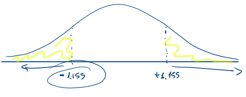
$$\begin{cases} H_0: \pi = \pi_0 = 0.25 \\ H_1: \pi \neq \pi_0 = 0.25 \end{cases}$$

$$\hat{\pi} = \frac{20}{100} = 0.20$$

$\boxed{B}$

$$\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}} \sim N(0,1)$$

$$z_{obs} = \frac{0.2 - 0.25}{\sqrt{\frac{0.25(1-0.25)}{100}}} = -1.155$$




$$\text{Pvalue} = P(|Z| > |z_{obs}|) = 2P(Z > 1.155) = 0.248$$

# TEST PER DUE CAMPIONI

$x_{1A}, x_{2A}, \dots, x_{n_A A}$ $n_A$ VC IID

$$x_{iA} \sim N(\mu_A; \sigma_A^2)$$

$x_{1B}, x_{2B}, \dots, x_{n_B B}$ $n_B$ VC IID

$$x_{iB} \sim N(\mu_B; \sigma_B^2)$$

TEST UNILAT.

$$\begin{cases} H_0: \mu_A = \mu_B \\ H_1: \mu_A > \mu_B \end{cases} \qquad \begin{cases} H_0: \mu_B = \mu_A \\ H_1: \mu_B < \mu_A \end{cases}$$

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TEST BILAT

$$\begin{cases} H_0: \mu_A = \mu_B \\ H_1: \mu_A \neq \mu_B \end{cases}$$

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$$\hat{\mu}_A = \frac{1}{n_A} \sum_{i=1}^{n_A} x_{iA} \sim N\left(\mu_A; \frac{\sigma_A^2}{n_A}\right)$$

$$\hat{\mu}_B = \frac{1}{n_B} \sum_{i=1}^{n_B} x_{iB} \sim N\left(\mu_B; \frac{\sigma_B^2}{n_B}\right)$$

$$\hat{\mu}_A - \hat{\mu}_B \sim N\left(\mu_A - \mu_B; \frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}\right)$$

$$\frac{(\hat{\mu}_A - \hat{\mu}_B) - (\mu_A - \mu_B)}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} \sim N(0,1)$$

sotto $\mu_A = \mu_B \Rightarrow \mu_A - \mu_B = 0$

sotto $H_0$

$$\frac{\hat{\mu}_A - \hat{\mu}_B}{\sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}} \sim N(0,1)$$

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Ipotesi di OMOGENEITÀ

$$\sigma_A^2 = \sigma_B^2 = \sigma^2$$

Ipotesi di ETEROGENEITÀ

$$\sigma_A^2 \neq \sigma_B^2$$

$$\begin{cases} H_0: \sigma_A^2 = \sigma_B^2 \\ H_1: \sigma_A^2 \neq \sigma_B^2 \end{cases}$$