PROB.

$0 \leq P(A) \leq 1$; $P(\emptyset) = 0$, $P(\Omega) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

CASO SP SE $A \cap B = \emptyset$ regola SOMMA

$\Rightarrow P(A \cap B) = P(\emptyset) = 0$

$\Rightarrow P(A \cup B) = P(A) + P(B) - 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \neq P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

CASO PARTICOLARE

SE A e B SONO INDIP.

$\Rightarrow P(A \cap B) = P(A)P(B)$

REGOLA COMPLEMENTO

$$P(\bar{A}) = 1 - P(A)$$

 2 ECR

1. PROB 1B e 1N nell'ordine

E_1 = "Bianca alla prima estr."
E_2 = "Bianca alla seconda estr."

"prima bianca e poi nera" $= E_1 \cap \bar{E}_2$

$$P(E_1 \cap \bar{E}_2) = P(E_1)P(\bar{E}_2) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$$

---

2. Prob. di 1B e 1N NON IMPORTA L'ORDINE

"1B e 1N" $= (E_1 \cap \bar{E}_2) \cup (\bar{E}_1 \cap E_2)$

$$P(\text{"1B e 1N"}) = P((E_1 \cap \bar{E}_2) \cup (\bar{E}_1 \cap E_2))$$
$$= P(E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2)$$
$$= P(E_1)P(\bar{E}_2) + P(\bar{E}_1)P(E_2)$$
$$= \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} + \frac{1}{3}$$
$$= \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$$

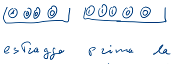
$n=2$ ESR

$$P(E_1 \cap \bar{E}_2) = P(E_1)P(\bar{E}_2|E_1) = \frac{1}{3} \times \frac{2}{2} = \frac{1}{3}$$

$$P(\bar{E}_1 \cap E_2) = P(\bar{E}_1)P(E_2|\bar{E}_1) = \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} = \frac{1}{3}$$

$$P((E_1 \cap \bar{E}_2) \cup (\bar{E}_1 \cap E_2)) = P(E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2)$$
$$= P(E_1)P(\bar{E}_2|E_1) + P(\bar{E}_1)P(E_2|\bar{E}_1)$$
$$= \frac{1}{3} + \frac{1}{3}$$
$$= \frac{2}{3}$$

A e B



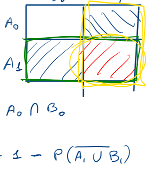
estraggo prima da A e poi DA B P("2 Biglietti ①")

A_0 : "esce 0 da A"
A_1 : "esce 1 da A"
B_0 : "esce 0 da B"
B_1 : "esce 1 da B"

$$P(A_1 \cap B_1) = P(A_1)P(B_1) = \frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$$

ALMENO 1 BIGLIETTO ①

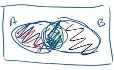
$$P(A_1 \cup B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1)$$
$$= \frac{1}{4} + \frac{3}{5} - \frac{3}{20}$$
$$= \frac{7}{10}$$



$\overline{A_1 \cup B_1} = A_0 \cap B_0$

$$P(A_1 \cup B_1) = 1 - P(\overline{A_1 \cup B_1})$$
$$= 1 - P(A_0 \cap B_0)$$
$$= 1 - P(A_0)P(B_0)$$
$$= 1 - \frac{3}{4} \times \frac{2}{5} = \frac{7}{10}$$

"Almeno 1 uno" = $A_1 \cup B_1$



Ω

$$A_1 \cup B_1 = (A_1 \cap B_0) \cup (A_1 \cap B_1) \cup (A_0 \cap B_1)$$

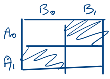
$$P(A_1 \cup B_1) = P(A_1 \cap B_0) + P(A_1 \cap B_1) + P(A_0 \cap B_1)$$

$$= P(A_1)P(B_0) + P(A_1)P(B_1) + P(A_0)P(B_1)$$

$$= \frac{1}{4} \times \frac{2}{5} + \frac{1}{4} \times \frac{3}{5} + \frac{3}{4} \times \frac{3}{5} = \frac{14}{20} = \frac{7}{10}$$

"somma uguale 1" = $(A_0 \cap B_1) \cup (A_1 \cap B_0)$

$$P(\text{"somma} = 1\text{"}) = P(A_0 \cap B_1) + P(A_1 \cap B_0)$$

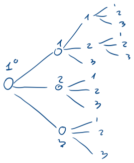


$$= P(A_0)P(B_1) + P(A_1)P(B_0)$$

$$= \frac{1}{4} \times \frac{2}{5} + \frac{3}{4} \cdot \frac{3}{5} = \frac{11}{20}$$

1	1, 1, 1
2	1, 1, 2
⋮	
9	1, 1, 9
10	1, 2, 1
⋮	
18	1, 2, 9

9^3



$9^3 = 729$

C_i = apro alla combinazione i, $i = 1, \dots, 5$

P("APRIRE ENTRO IL 5° TENTATIVO")

$$= P(C_1 \cup (\bar{C}_1 \cap C_2) \cup (\bar{C}_1 \cap \bar{C}_2 \cap C_3) \cup (\bar{C}_1 \cap \bar{C}_2 \cap \bar{C}_3 \cap C_4) \cup (\bar{C}_1 \cap \bar{C}_2 \cap \bar{C}_3 \cap \bar{C}_4 \cap C_5))$$

$$= P(C_1) + P(\bar{C}_1 \cap C_2) + \dots + P(\bar{C}_1 \cap \bar{C}_2 \cap \dots \cap C_5)$$

$$= \frac{1}{729} + \frac{728}{729} \cdot \frac{1}{729} + \frac{728}{729} \times \frac{728}{729} \frac{1}{729} + \left(\frac{728}{729}\right)^3 \frac{1}{729} + \left(\frac{728}{729}\right)^4 \frac{1}{729}$$

$$= 0.00684$$

$\overline{\text{"APRO ENTRO 5 PROVE"}}$ = "NON APRO IN 5 PROVE"

$$P(B) = 1 - P(\bar{B})$$

$$P(\bar{B}) = P(\bar{C}_1 \cap \bar{C}_2 \cap \bar{C}_3 \cap \bar{C}_4 \cap \bar{C}_5)$$

$$= P(\bar{C}_1)P(\bar{C}_2) \cdot \dots \cdot P(\bar{C}_5)$$

$$= \frac{728}{729} \times \frac{728}{729} \times \dots \times \frac{728}{729}$$

$$= \left(\frac{728}{729}\right)^5$$

$$P(B) = 1 - \left(\frac{728}{729}\right)^5 = 0.00684$$

$$P(B) = 1 - P(\bar{B})$$

$$P(\bar{B}) = P(\bar{C}_1 \cap \bar{C}_2 \cap \bar{C}_3 \cap \bar{C}_4 \cap \bar{C}_5)$$

$$= P(\bar{C}_1)P(\bar{C}_2 | \bar{C}_1)P(\bar{C}_3 | \bar{C}_1 \cap \bar{C}_2) \times P(\bar{C}_4 | \bar{C}_1 \cap \bar{C}_2 \cap \bar{C}_3)P(\bar{C}_5 | \bar{C}_1 \cap \dots \cap \bar{C}_4)$$

$$= \frac{\cancel{728}}{729} \times \frac{\cancel{727}}{\cancel{728}} \times \frac{\cancel{726}}{\cancel{727}} \times \frac{\cancel{725}}{\cancel{726}} \times \frac{724}{\cancel{725}}$$

$$= \frac{724}{729}$$

$$P(B) = 1 - P(\bar{B}) = 1 - \frac{724}{729} = 0.00686$$

20 25

A = "PROMOSSO 1° APPELLO"

= "CONOSCE 3 RISPOSTE"

$= S_1 \cap S_2 \cap S_3$

$$P(\text{"PASSARE 1° APP."}) = P(S_1 \cap S_2 \cap S_3)$$

$$= P(S_1)\,P(S_2 \mid S_1)\,P(S_3 \mid S_1 \cap S_2)$$

$$= \frac{20}{25} \times \frac{19}{24} \times \frac{18}{23} = 0.496$$

B = "PASSARE ENTRO IL 3° APP."

$$B = A_1 \cup (\bar{A}_1 \cap A_2) \cup (\bar{A}_1 \cap \bar{A}_2 \cap A_3)$$

$\bar{B}$ = "NON PASSO L'ESAME 3 APP"

$$\bar{B} = \bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3$$

$$P(\bar{B}) = P(\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3)$$

$$= P(\bar{A}_1)\,P(\bar{A}_2)\,P(\bar{A}_3)$$

$$= (1-0.496)(1-0.496)(1-0.496)$$

$$= (1-0.496)^3$$

$$P(B) = 1 - P(\bar{B})$$

$$= 1 - (1-0.496)^3 = 0.87171$$

		1/6	1/6	1/6	1/6	1/6	1/6	
		1	2	3	4	5	6	Ω
1/6	1	(1,1) 2	(1,2) 3	(1,3) 4	5	6	7	
1/6	2	3	4	5	6	7	8	
1/6	3	4	5	6	7 1/36	8	9	
1/6	4	5	6	7	8	9 1/36	10	
1/6	5	6	7	8	9	10	11	
1/6	6	7	8	9	10	11	12	

$$P(\boxed{4} \cap \boxed{2}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$$P(\text{"SOMMA} = 4\text{"}) = P(\boxed{1} \cap \boxed{3}) + P(\boxed{2} \cap \boxed{2}) + P(\boxed{3} \cap \boxed{1})$$

$$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{3}{36}$$